Closed timelike curves in general relativity

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Abstract

Many solutions of Einstein's field equations contain closed timelike curves (CTC). Some of these solutions refer to ordinary materials in situations which might occur in the laboratory, or in astrophysics. It is argued that, in default of a reasonable interpretation of CTC, general relativity does not give a satisfactory account of all phenomena within its terms of reference.

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In general relativity a timelike curve in spacetime represents a possible path of a physical object or an observer. Normally such a curve will run from past to future, but in some spacetimes timelike curves can intersect themselves, giving a loop, or a *closed timelike curve* (CTC). CTCs suggest the possibility of time-travel with its well-known paradoxes.

The first spacetime in which CTCs were noticed was that of Gödel. [1]. This represents a rotating universe without expansion, and requires a negative cosmological constant. As a model of physical reality it can therefore be dismissed because it is unlike the universe we live in. Another simple spacetime containing CTCs is that of van Stockum [2] which represents a cylinder of rigidly rotating dust; however, the cylinder is of infinite length so it could not be realised in practice.

Since these early discoveries other spacetimes containing CTCs have been found. Nearly all of these have been regarded as of merely theoretical interest because of some non-physical feature in their composition. [1] Recently, however, there have been published some solutions of Einstein's equations containing CTCs and representing physical situations which in principle could be reproduced in the laboratory, or might occur in astrophysics.

One such solution represents two spinning particles of masses m_1, m_2 and constant angular momenta h_1, h_2, their spins both parallel to their line of separation. The particles are fixed on the z-axis at $z = \pm b$. They are not supposed to represent black holes: they could be copper spheres in a laboratory.

[1] An exception is the Kerr-Newman solution, which is asymptotically flat and contains CTCs. However, these are unlikely to be realised in astrophysics or in the laboratory.

The set-up is axially symmetric and assumed independent of time. so one may use the metric

$$ds^2 = -f^{-1}[\exp\nu(dz^2 + dr^2) + r^2 d\theta^2] + f(dt - wd\theta)^2, \tag{1}$$

f, ν, w being functions of z and r only. The coordinates will be numbered

$$x^1 = z,\ x^2 = r,\ x^3 = \theta,\ x^4 = t,$$

and their ranges are

$$-\infty < z < \infty,\ 0 \le r,\ 0 \le \theta \le 2\pi,\ -\infty < t < \infty,$$

$\theta = 0$ and $\theta = 2\pi$ being identified.

Reference [3] contains details of the vacuum Einstein equations and an approximate solution up to terms quadratic in the parameters m_1, m_2, h_1, h_2. One finds, as expected, that there is a singularity between the particles representing a strut supporting them against their mutual gravitation. However, the important expression for my purposes here is

$$g_{33} = -f^{-1}[r^2 - f^2 w^2].$$

If θ is a spacelike coordinate this should be negative, but it turns out that on the axis of symmetry $r = 0$ one can arrange this either between the particles or outside them, but not both unless the parameters satisfy the relationship

$$m_1 h_2 + m_2 h_1 = 0. \tag{2}$$

Let us suppose for the moment that (2) is not satisfied, and that there is a region D between the particles in which $g_{33} > 0$. Then θ is in D a timelike coordinate which is cyclic because the hypersurfaces $\theta = 0$ and $\theta = 2\pi$ are identified. Hence the spacetime contains CTC.

Eqn (2) compounds the mystery. It can be written in the very simple form

$$a_1 + a_2 = 0,$$

where a_1 and a_2 are the angular momenta per unit mass of the two particles. There is no obvious reason why CTC should be absent in this case.

There are other simple physical systems in which CTC cannot be avoided. One consists of a static magnetic dipole μ and a static electric charge e placed on dipole's axis. They have masses m_1 and m_2 and they need a strut between them to counterbalance gravitation. The set-up looks static, but in fact one cannot solve the Einstein-Maxwell equations with an axially symmetric static metric - one needs the stationary (1). In [4] I found an approximate solution for this system, correct to the quadratic terms in the parameters m_1, m_2, e, μ. Once again, positive values of g_{33}, and therefore CTC, must occur near the axis either between the particles or outside them.

These examples are approximate solutions of the field equations, but somewhat similar exact solutions are known. One arises from the Perjes-Israel-Wilson (PIW) solutions [5][6] of the Einstein-Maxwell equations. The PIW metrics are generalisations of the Papapetrou-Majumdar ones: they represent sources consisting of spinning masses bearing electric charge and magnetic dipole moment, such that, in relativistic units, the mass of each particle equals its charge, and its spin is equal to its magnetic moment. It was shown long ago [7] that two PIW particles in axi-symmetric configuration engender CTC unless their parameters satisfy a relation like (2).

Another example, shortly to be published [8], is an exact solution for a finite spinning rod. The solution can be constructed from the Papapetrou class [9] which represents spinning massless objects. It too has CTC. One can add mass as a perturbation to the exact solution and the CTC persist. CTC also exist in the vacuum spinning C-metric [10].

These solutions refer to ordinary materials that might occur in the laboratory, or in astrophysics. They are asymptotically flat. What can CTC mean in these cases? We surely cannot believe that, for example, a charge and a magnet constitute a time machine! I have suggested [3] that, where it occurs between particles, a CTC region represents a *torsion singularity* constraining their spin, but this explanation does not seem to apply to the case of the spinning rod.

I believe there is an urgent need to find a convincing physical interpretation of CTC. They can no longer be dismissed as curiosities occurring in non-physical solutions. We now know that there are simple physical situations, such as that of the charge and the magnet, within the terms of reference of general relativity, of which the theory as currently understood does not give a satisfactory account.

References

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